Journal of Sound and Vibration (2002) **255**(3), 549–554
doi:10.1006/jsvi.2001.4179, available online at http://www.idealibrary.com on IDEAL®

IN-SERVICE IDENTIFICATION OF NON-LINEAR DAMPING FROM MEASURED RANDOM VIBRATION

D. V. IOURTCHENKO

Mechanical Engineering Department, University of Miami, P.O. Box 248294, *Coral Gables, FL* 33124-0624, *U.S.A. E-mail*: *daniil@miami.edu*

AND

M. F. DIMENTBERG

Mechanical Engineering Department, Worcester Polytechnic Institute, 100 *Institute Road, Worcester, MA,* 01609, *U.S.A.*

(*Received* 13 *August* 2001, *and in final form* 14 *November* 2001)

A non-linearly damped single-degree-of-freedom (s.d.o.f.) system under broadband random excitation is considered. A procedure for in-service identification of the damping characteristic from measured stationary response is described. The procedure is based on the stochastic averaging method. The explicit analytical solution is obtained for the integral equation, which relates the desired damping characteristics to the apparent force in the shortened equation for the slowly varying response amplitude, and thus to the measured probability density of the amplitude. The approach is of a non-parametric nature, which makes it convenient for testing hypotheses of damping mechanisms from measured random vibration data. Extensive results of numerical tests for the procedure are presented.

1. INTRODUCTION

Problems of identification of systems' parameters are very important in engineering. There are three different types of identification problems, such as on-line, in-service and in-tests. On-line identification, or one "in-real time", implies permanent monitoring of system's performance and applying control force(s), if necessary, based on measured data. Feedback control is a good example of "real-time" monitoring. On-line identification may help in timely detecting undesirable failures due to fatigue or crack propagation, for instance. In-service identification is based on signal processing of measured response to "natural" excitation, which may not be controlled and/or measurable. The on-line feedback control, however, is not required in this case. Thus, off-line signal processing may be used, particularly for the case of random excitation, to estimate statistical characteristics of the response that can be used for identification. As for identification in special tests, a controlled input excitation can be applied, such as that in a simple hammer test or in shaker test with harmonic excitation.

Solution to an "inverse", or identification problem may imply, for example, development of algorithm for evaluating unknown energy dissipation mechanism from response measurements for a given structure. Such an algorithm should be strongly dependent on the type of identification. This paper is focused specifically on the in-service type of

0022-460X/02/$35.00

identification for the case, where a broadband random excitation is the source of the available response signal.

A number of identification methods have been proposed in last decades, such as those due to references [1–4] etc. The method for in-tests identification of a non-linear single-degree-of-freedom (s.d.o.f.) system with harmonic excitation, as introduced by Dimentberg [5], should be mentioned here. It is based on the harmonic balance approximation for the response, which expresses steady state response amplitude and phase as integrals of the original non-linear system's characteristics over the period of the corresponding linear systems. In the "inverse" problem of identification these relations are actually integral equations of the first kind. More specifically, as shown in reference [5], each of these equations is the Schlomilch's integral equation, which has an exact analytical solution [6]. This solution provided explicit expression for the unknown non-linear damping characteristics in terms of measured response amplitude and phase. The corresponding identification method has been successfully implemented and also extended to the case of parametric excitation by Iourtchenko *et al.* [7]. The accuracy of the method has been demonstrated, it depends on closeness of excitation frequency to the system's natural frequency. This conclusion is clear, as long as the harmonic balance approximation is particularly accurate in case of resonant oscillations. However, this approximation is advantageous for accuracy of the resulting identification method, since it does not require differentiation of the rapidly varying response signal.

This paper represents a modification of the above method for the case of in-service identification with broadband random excitation being the source of the measured response. The stochastic averaging procedure leads to "shortened" stochastic differential equations for slowly varying steady state response's amplitude and phase. An analytical solution is known for the corresponding Fokker–Plank–Kolmogorov (FPK) equation for probability density function (p.d.f.) of steady state response amplitude [8, 9], [5]. As long as its p.d.f. is available for the measured response signal of the system to be identified, the above analytical solution may be regarded as the Schlomilch integral equation for the unknown damping characteristics. Thus, the explicit solution for the latter is obtained, provided the excitation intensity is known. Numerical simulation test results for the method are presented in this paper for three cases of damping: linear viscous damping, dry-friction damping and linear combination of these two.

There is a significant difference between these two methods. In contrast to the method with an external deterministic excitation, which requires several experimental runs to recover a damping characteristic, one with a random input excitation requires just one long simulation, which gives "all" possible stochastic information about the system's response (the output process is assumed to be ergodic). Therefore, the method with random excitation is an in-service type, whereas the method with harmonic excitation is an in-test one.

2. PROBLEM FORMULATION

A s.d.o.f. system under zero mean Gaussian white-noise excitation is considered

$$\ddot{x} + H(\dot{x}) + \Omega^2 x = \sigma\xi(t), \tag{1}$$

where H is a non-linear, odd function and σ^2 is an intensity of white noise $\xi(t)$. Actually, as long as the system is considered to be lightly damped, the white-noise approximation may be somewhat relaxed. Namely, $\xi(t)$ may be regarded just as a broadband random process with spectral density $S(\omega)$, and $\sigma^2 = 2\pi S(\Omega)$. The process's spectral density may be considered in this case as a constant in the vicinity of the system's natural frequency. Thus, the latter expression simply shows that the noise intensity σ^2 is proportional to a value of spectral density of a broadband process at the system's natural frequency. According to the stochastic averaging method, slowly varying amplitude and phase are introduced as new state variables using the transformation

$$x = A\cos\theta, \qquad \dot{x} = -\Omega A\sin\theta, \qquad \theta = \Omega t + \phi \tag{2}$$

which is substituted into equation (1). Then a set of two Stratonovich or physical stochastic differential equations is obtained

$$\dot{A} = H(-A\Omega\sin\theta)\frac{\sin\theta}{\Omega} - \frac{\sin\theta}{\Omega}\sigma\xi(t), \qquad \dot{\phi} = H(-A\Omega\sin\theta)\frac{\cos\theta}{A\Omega} - \frac{\cos\theta}{A\Omega}\sigma\xi(t). \tag{3}$$

According to the stochastic averaging method [8, 9], the RHSs of these SDEs may be approximated by their averaged-over-the-period expressions. The averaging is performed over rapidly varying time, with slowly varying amplitude and phase being kept constant. By adding Wong–Zakai corrections one may obtain the following "shortened" Ito SDE for the response amplitude

$$\dot{A} = -h(A) + B_\xi/A + 2B_\xi\zeta(t), \quad h(A) = \frac{1}{2\pi\Omega}\int_0^{2\pi} H(A\Omega\sin\theta)\sin\theta\,\mathrm{d}\theta, \qquad B_\xi = \frac{\sigma^2}{4\Omega^2}. \tag{4}$$

A stationary p.d.f. $p(A)$ of the amplitude satisfies the corresponding FPK equation

$$\frac{\mathrm{d}}{\mathrm{d}A}\{[-h(A) + B_\xi/A]p\} = \frac{\mathrm{d}^2}{\mathrm{d}A^2}\{B_\xi p\} \tag{5}$$

and may be found as [8, 9]

$$p(A) = 2CA\mathrm{e}^{-G(A)}, \quad \text{where } \frac{\mathrm{d}}{\mathrm{d}A}G(A) = \frac{h(A)}{B_\xi}, \tag{6}$$

where C is a normalizing constant. An accurate interpolation of a p.d.f. of the response's amplitude, $p(A)$ from experimental data is very important. The authors found it more convenient, to work with another, new variable $V = A^2$. Equation (6) is transformed then to the p.d.f. $p(V)$ of the new variable V, and the unknown function $G(\sqrt{V})$ may be expressed accordingly as

$$p(V) = C\mathrm{e}^{-G(\sqrt{V})} \quad \text{and} \quad G(\sqrt{V}) = \ln C - \ln p(V). \tag{7}$$

Thus, if $p(A)$ is a known (measured) function, then the corresponding function $h(A)$, can be used to recover an unknown, non-linear damping characteristic, provided that the excitation intensity is known. The available solution to the Schlomilch's integral equation is used to this end. Namely, the integral equation

$$f(v) = \frac{2}{\pi}\int_0^{\pi/2}\Psi(v\sin\varphi)\,\mathrm{d}\varphi \tag{8}$$

with known function $f(v)$ has the explicit solution [6]:

$$\Psi(v) = f(0) + v\int_0^{\pi/2} f'(v\sin\varphi)\,\mathrm{d}\varphi. \tag{9}$$

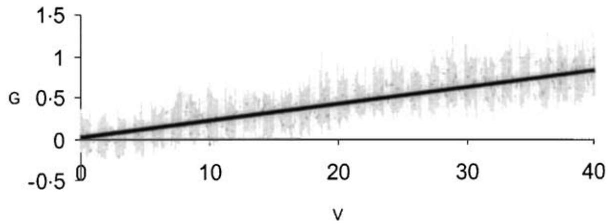

Figure 1. Logarithm of normalized stationary p.d.f. of response amplitude and its interpolation: linear.

Relation (4) for $h(A)$, with the latter function being known from the measured response p.d.f., may be reduced to form (8), where

$$v = A\Omega, \quad f(v) = \Omega h(v/\Omega)v, \qquad \Psi(v \sin\varphi) = uH(u), \quad u = v\sin\varphi.$$

Thus, using solution (9) the unknown damping characteristic may be obtained as

$$H(v) = \Omega \int_0^{\pi/2} \{h(v\sin\varphi) + v\sin\varphi\ h'(v\sin\varphi)\}\,\mathrm{d}n. \tag{10}$$

3. IMPLEMENTATION AND RESULTS

To demonstrate how the foregoing method works and to illustrate its accuracy, three simple models of damping are considered: linear viscous friction, dry friction and combination of both. Numerical simulation of equation (1) has been done to acquire the required response data using fourth order scheme Runge–Kutta, with unit intensity of white noise. Statistical information of response amplitude was collected for $6500T$, where $T = 2\pi/\Omega$ is the system's natural period.

Consider first the case of linear viscous friction. The results were obtained from numerical simulation for $H = \alpha\dot{x}$, $\alpha = 0{\cdot}02$ and $\alpha/\Omega = 0{\cdot}02$. Figure 1 demonstrates the logarithm of the normalized stationary p.d.f. of the response's amplitude (7) and its interpolation.

Taking into account equation (6) for $h(A)$, an unknown damping characteristic is recovered

$$G(\sqrt{V}) = 0{\cdot}0204V = 0{\cdot}0204A^2, \qquad h(A) = \frac{0{\cdot}0204}{2}A, \quad h(v) = \frac{0{\cdot}0204}{2}v,$$

$$H(v) = \int_0^{\pi/2} \frac{0{\cdot}0408 v\sin\varphi}{2}\,\mathrm{d}\varphi = 0{\cdot}0204v. \tag{11}$$

Figure 1 and equation (11) show that the damping characteristic, as restored according to the present approach, is linear indeed, whereas the assigned (exact) value of the damping factor is found with 2% accuracy. Such a high accuracy could be expected for the linear system.

As a second example, a system with (non-linear) dry friction damping is considered, where $H = R\,\mathrm{sgn}\,\dot{x}$, $R = 0{\cdot}1$. The result of numerical simulation is shown in Figure 2.

The interpolation of data results in the following estimates for $G(A)$, $h(A)$ and dissipation characteristics $H(v)$ accordingly:

$$G(\sqrt{V}) = 0{\cdot}2821\sqrt{V} = 0{\cdot}2821A, \quad h(A) = 0{\cdot}2821B_\xi = \frac{0{\cdot}2821}{4} = Const,$$

$$H(v) = \Omega \int_0^{\pi/2} \frac{0{\cdot}2821}{4}\,\mathrm{d}\varphi = 0{\cdot}11. \tag{12}$$

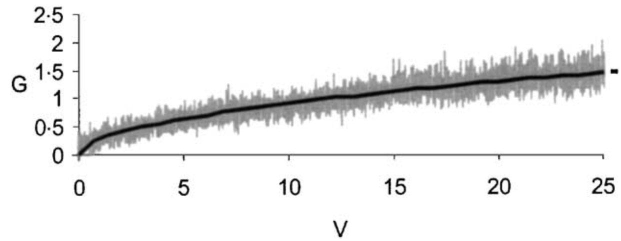

Figure 2. Logarithm of normalized stationary p.d.f. of response amplitude and its interpolation: dry friction.

As it can be seen from equation (12), the absolute error in the estimate of the dry-friction coefficient is higher for this case—about 10%. However, this error can still be considered as an acceptable one for in-service measurements in engineering practice.

Finally, the case of a combined non-linear damping characteristic, consisting of both linear and dry-friction parts, is considered, with the same values of R and α. Numerical simulation produced the results

$$G(\sqrt{V}) = 0{\cdot}0181V + 0{\cdot}2614\sqrt{V}, \qquad h(v/\Omega) = \frac{0{\cdot}0181}{2}v + \frac{0{\cdot}2614}{4},$$

$$\mathrm{H}(v) = \int_0^{\pi/2}\left\{\frac{0{\cdot}0181}{2}v\sin\varphi + \frac{0{\cdot}2614}{4}\right\}\mathrm{d}\varphi + \int_0^{\pi/2} v\sin\varphi\left\{\frac{0{\cdot}0181}{2}\right\}\mathrm{d}\varphi = 0{\cdot}0181v + 0{\cdot}103. \tag{13}$$

Results (13) indicate that the estimate of dry-friction coefficient has improved significantly, whereas the absolute error in the estimate of the linear damping coefficient increased up to 8%.

4. CONCLUSIONS

The in-service identification method has been developed and implemented for randomly excited non-linear systems. The method does not require continuous measurements of the excitation, as long as the latter is known to be broadband; the intensity level of the excitation should be known only. As long as the method relies on the averaging over the period of the non-linear damping function, it should be regarded as being approximate. On the other hand, operation with slowly varying state variables provides also a very important advantage. Namely, the method should be highly robust with respect to measurement errors, as long as no direct operations with rapidly varying original signals are required. The ultimate verification of accuracy should be obtained, of course, in special computer runs with artificially introduced simulated measurement errors.

The method can be clearly extended to multi-degree-of-freedom systems with well-separated (coupled) natural frequencies. Applying it to the band-pass-filtered response signal, with the filters being tuned to various peaks of the response spectral density, should provide the corresponding modal damping functions.

REFERENCES

1. K. S. Mohammad, K. Worden and G. R. Tomlinson 1992 *Journal of Sound and Vibration* **152**, 471–499. Direct parameter estimation of linear and non-linear structures.

2. J. V. TOMME 1988 *13th International Seminar on Modal Analysis*. Evaluation of damping measurements in materials and structures.
3. B. J. LAZAN 1968 *Damping of Materials and Members in Structural Mechanics*. Oxford: Pergamon Press.
4. J. NAPRSTEK 1999 *Stochastic Structural Dynamics*. Brookfield: A. A. Balkema. *Proceedings of the 4th International Conference on Stochastic Structural Dynamics*. Identification of linear and non-linear dynamic systems with parametric noises.
5. M. F. DIMENTBERG 1962 *Machanica Tverdogo Tela* **2**, 32–34. Determination of a non-linear damping function from forced vibration test of a SDOF system (in Russian).
6. T. WHITTAKER and G. N. WATSON 1962 *A Course of Modern Analysis*. Cambridge: Cambridge University Press.
7. D. V. IOURTCHENKO, L. DUVAL and M. F. DIMENTBERG 2000 *Developments in Theoretical and Applied Mechanics. Proceedings of SECTAM XX Conference, Auburn University*. The damping identification for certain SDOF systems.
8. M. F. DIMENTBERG 1988 *Statistical Dynamics of Nonlinear and Time-Varying Systems*. Taunton: Research Study Press.
9. Y. K. LIN and G. Q. CAI 1995 *Probabilistic Structural Dynamics*. New York: McGraw-Hill.